A. Henriques, nov. 2005.

- Given a space X the *Yoneda functor* maps it to

$$\begin{array}{rcl} \mathrm{Y}(X) : \mathsf{Top} & \longrightarrow & \mathsf{Sets} \\ T & \mapsto & Hom(T,X). \end{array}$$

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- A *groupoid* is a category where all arrows are isomorphisms.

- Let X be a set. The groupoid $\mathrm{I}X$ has $Objects(\mathrm{I}X) = X$ and only identity arrows.

- Let G be a group. The groupoid $\mathrm{E}G$ has $Objects(\mathrm{E}G) = G$ and exactly one arrow between any two objects. Note that $\mathrm{E}G$ is equivalent to the trivial groupoid, and that it has a free action of G.

- The groupoid $\mathrm{B}G := \mathrm{E}G/G$ has only one object and G many arrows.

- Let G be a group acting on a set X. The *quotient groupoid* is given by $X /\!\!/ G := (X \times \mathrm{E}G)/G$. It has $Objects(X /\!\!/ G) = X$. Moreover, an arrow $x \to y$ is the same thing as a group element g such that $gx = y$.

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- A *stack* is a functor $F : \mathsf{Top} \to \mathsf{Gpds}$ such that for every space T and every open cover $\{U_i\}$ of T, each time we are given:
  - objects f_i in $F(U_i)$,
  - morphisms $\alpha_i : f_i \to f_j$ in $F(U_i \cap U_j)$,
  - making the diagram $\begin{array}{ccccc} & & f_j & & \\ & \nearrow^{\alpha_{ij}} & & \searrow^{\alpha_{jk}} & \\ f_i & & \xrightarrow{\alpha_{ik}} & & f_k \end{array}$ commute in $F(U_i \cap U_j \cap U_k)$,

then there exists an essentially unique choice of:
  - object f in $F(T)$,
  - morphisms $\beta_i : f \to f_i$ in $F(U_i)$,
  - making the diagram $\begin{array}{ccccc} & & f_i & & \\ & \nearrow^{\beta_i} & & \searrow^{\alpha_{ij}} & \\ f & & \xrightarrow{\beta_j} & & f_j \end{array}$ commute in $F(U_i \cap U_j)$.

- Example: The functor $\mathbb{Y}(X)$ given by $T \mapsto \mathrm{I}Hom(T,X)$ is a stack.

- Given a functor $F : \mathsf{Top} \to \mathsf{Gpds}$, its *stackification* F' is given by:
  - An object of $F'(T)$ is an open cover $\{U_i\}$ and a collection $f_i \in F(U_i)$, $\alpha_{ij} : f_i \to f_j$ as above.
  - A morphism $(U_i, f_j, \alpha_{ij}) \to (V_a, g_a, \beta_{ab})$ is a collection of morphisms $\gamma_{ia} : f_i \to g_a$ in $F(U_i \cap V_a)$

  making the diagram $\begin{array}{ccc} f_i & \xrightarrow{\alpha_{ij}} & f_j \\ {\scriptstyle\gamma_{ia}}\downarrow & & \downarrow{\scriptstyle\gamma_{jb}} \\ g_a & \xrightarrow{\beta_{ab}} & g_b \end{array}$ commute in $F(U_i \cap U_j \cap V_a \cap V_b)$.

- Let G be a topological group. Then $\mathbb{B}G$ is the stackification of the functor $T \mapsto \mathrm{B}Hom(T,G)$.

- Let G be a topological group acting on a topological space X. Then the *quotient stack* $[X/G]$ is the stackification of the functor $T \mapsto Hom(T,X) /\!\!/ Hom(T,G)$.