Hand in exercise AQFT - The statistics of an endomorphism

Let ρ be an endomorphism of $\mathcal{A}$, localized in an interval $I \in \mathscr{I}$. Choose an equivalent endomorphism ρ_0 localized in an interval I_0 with $\bar{I_0} \cap \bar{I} = \emptyset$ and let u be a local intertwiner in (ρ, ρ_0), that is, $u \in (\rho, \rho_0)_{\tilde{I}}$ with $I_0 \cup I \subset \tilde{I}$ and I_0 to the left (clockwise on the circle) of I in $\tilde{I}$.

Definition 1. ***The statistics operator*** *is defined as* $\epsilon := u^* \rho(u)$

Prove the following:

Proposition 1.

- $\epsilon \in (\rho^2, \rho^2)$
- *Defining* $\epsilon_i = \rho^{i-1}(\epsilon)$*, this gives rise to a representation of the Artin braid group:*

$$\epsilon_i \epsilon_{i+1} \epsilon_i = \epsilon_{i+1} \epsilon_i \epsilon_{i+1}, \quad \epsilon_i \epsilon_j = \epsilon_j \epsilon_i \text{ if } |i-j| \geqq 2$$

Definition 2. *The (unitary equivalence class of the) representation of the braid group that we obtain in this way is called* ***the statistics*** *of the superselection sector* ρ